Details that do not fit

(i) Form of expt^l C_V vs T graph
sharper than predicted – see sheet.

(ii) B-E theory predicts as $T \to 0$
all fluid $\longrightarrow$ condensate.

Neutron diffraction expts measure superfluid
as ~ 14% as $T \to 0$
though viscosity expts agree better with
B-E theory.

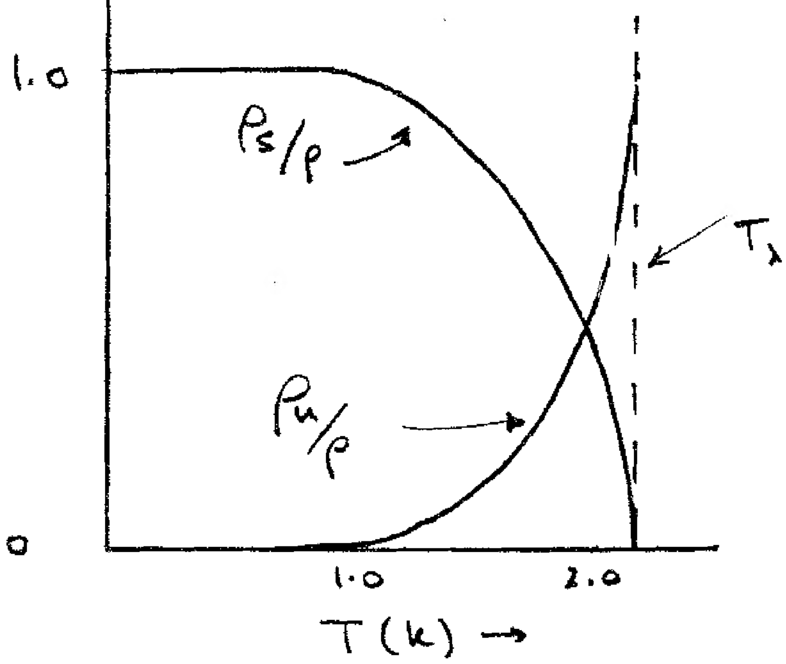
Reason for differences between experiment and
Bose – Einstein theory –

Interactions between He^4 atoms

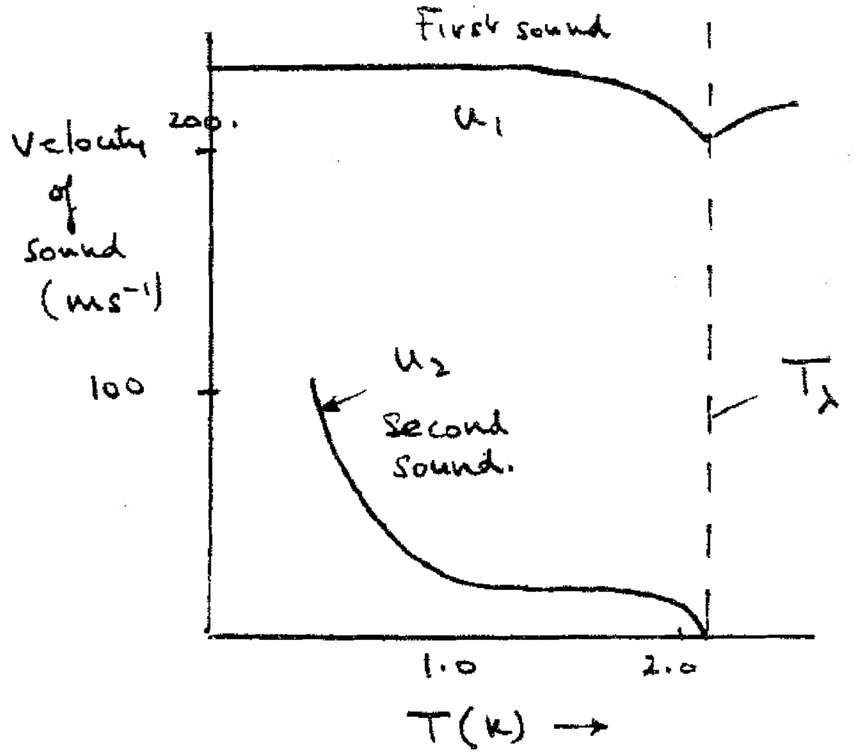
Boson interactions not strong but cannot be
treated with 'quasiparticle' theory like fermions.

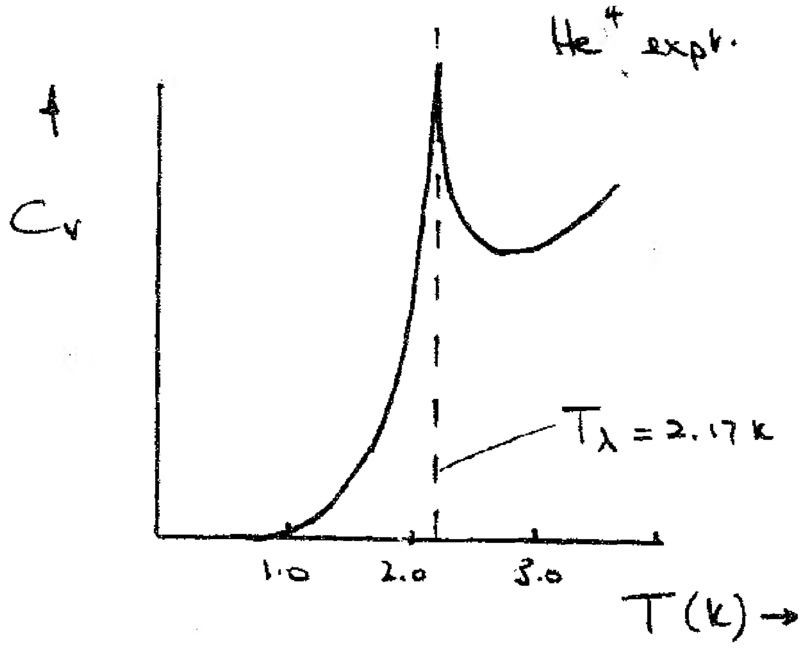
Liquid He II.

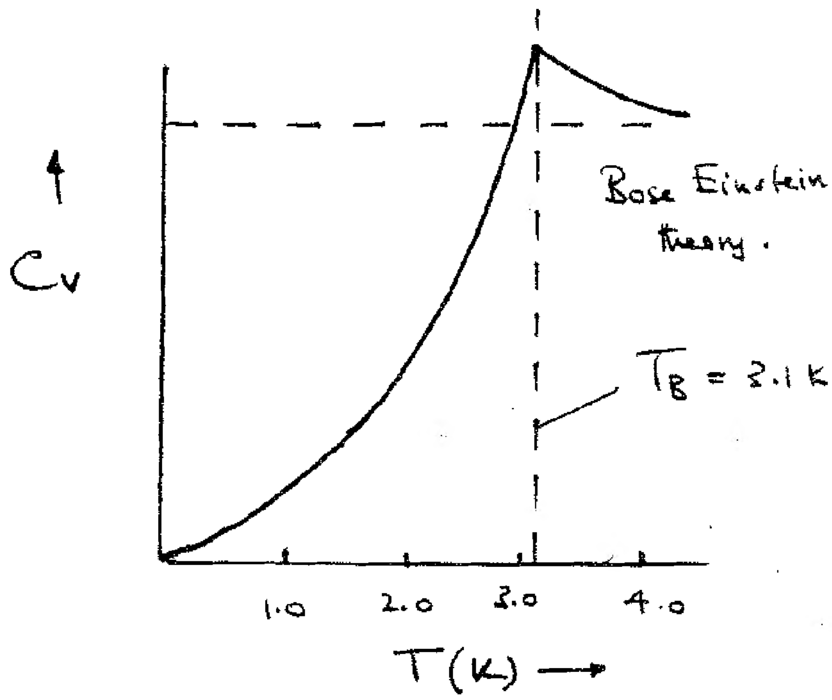
Ratio of superfluid and normal components from Andronikashvili's experiment.



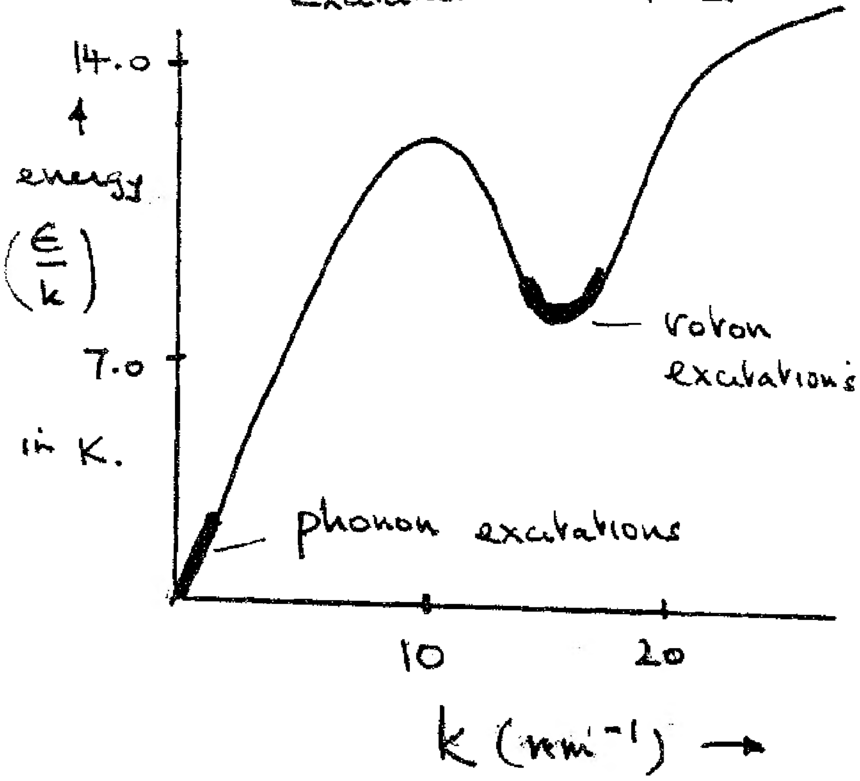
Velocities of sound vs temperature







Excitations in He II



Excitations in liquid He II

Measured by inelastic neutron scattering

Graph of ϵ vs k — see sheet.

Points.

(i) Whole curve measure by neutron experiment but thermally excited modes only in regions indicated

(ii) For ϵ, k small — excitations are like phonons.

For ϵ, k at dip in curve — excitations called rotons — involves quantised rotation but not simple picture.

Table Captions

Excitations in liquid He II

Measured by inelastic neutron scattering

Graph of ϵ vs k — see sheet.

Points.

(ii) For ϵ, k small — excitations are like phonons.

For ϵ, k at dip in curve — excitations called rotons — involves quantised rotation but not simple picture.

He^3 / He^4 liquid mixtures

Interesting because

(i) Example of near perfect Fermi gas

(ii) Are the working substance for He dilution refridgerator — standard way of cooling to ~ 5 mK.

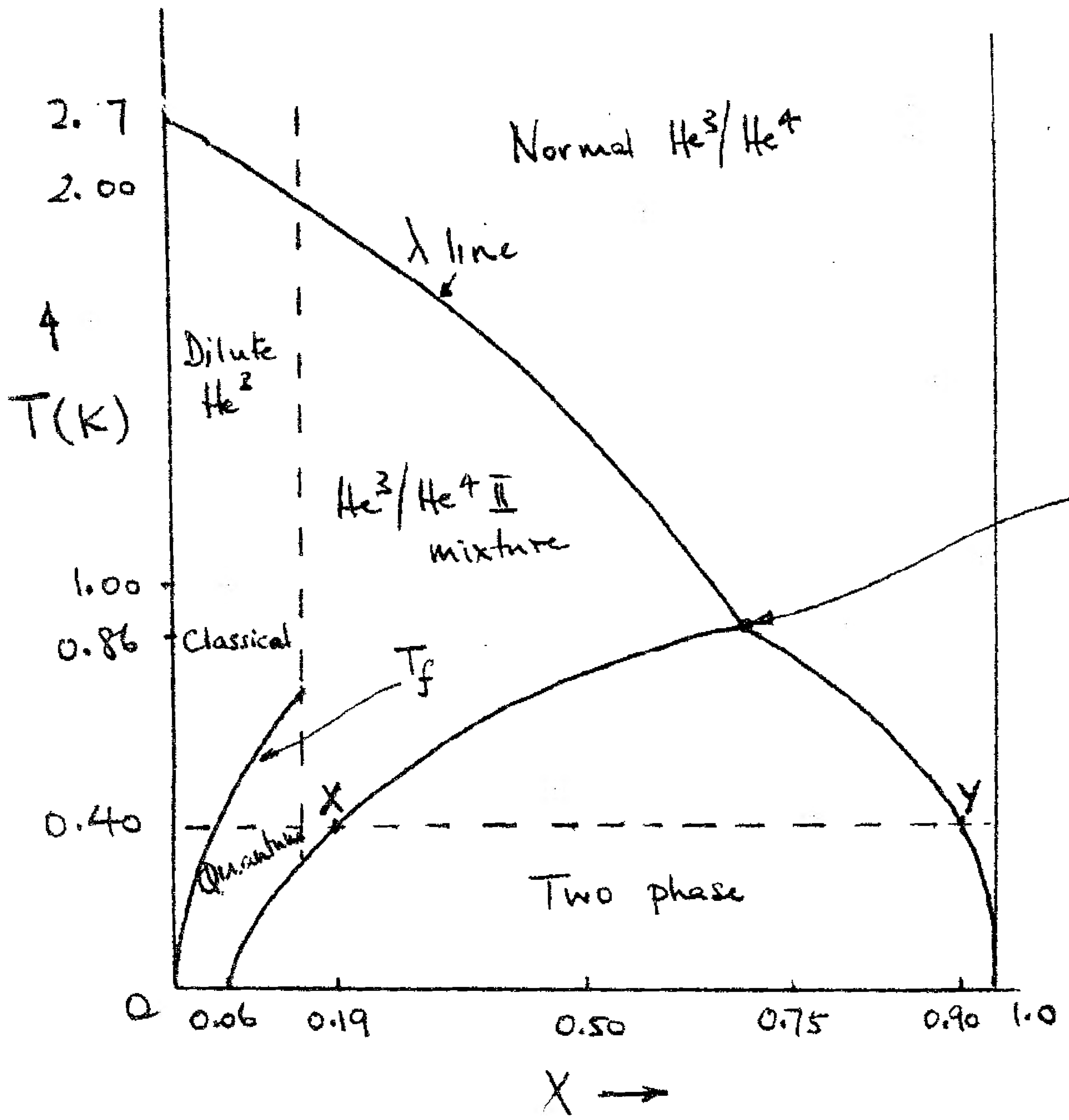
Main features related to phase diagram — see sheet.

Points

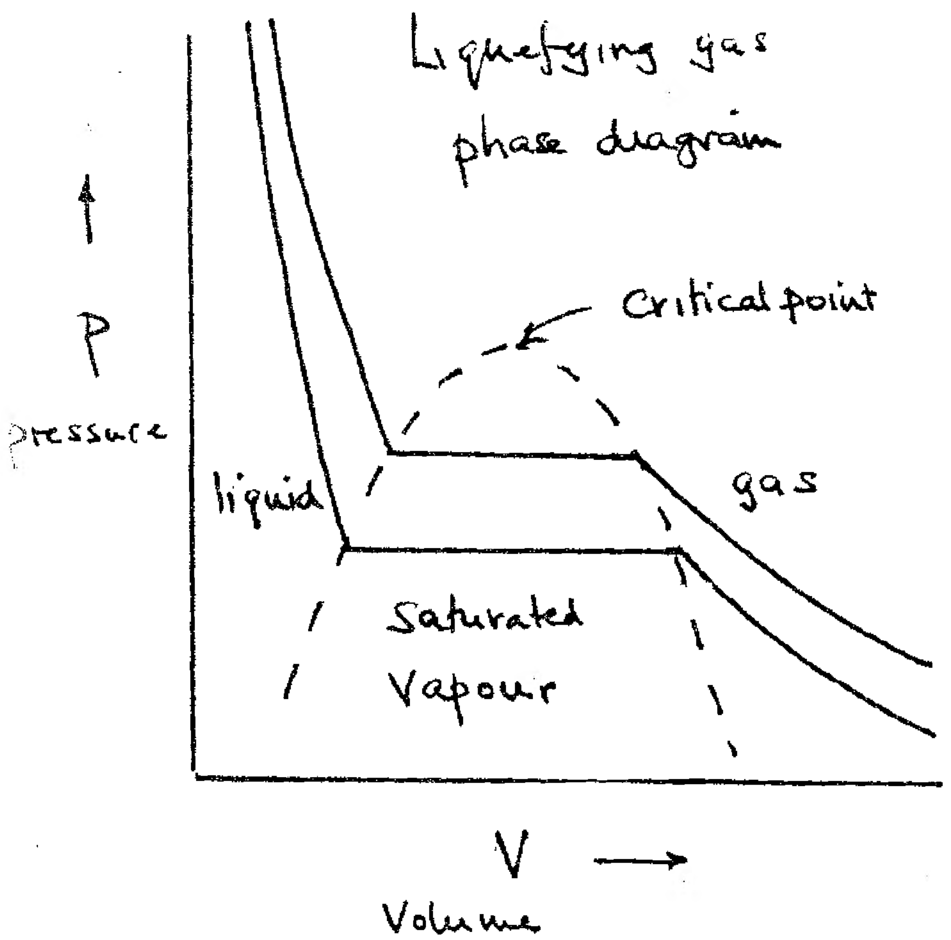
(i) Phase diagram shows state of He^3/He^4 liquid mixture in conditions defined by temperature and proportion X of He^3

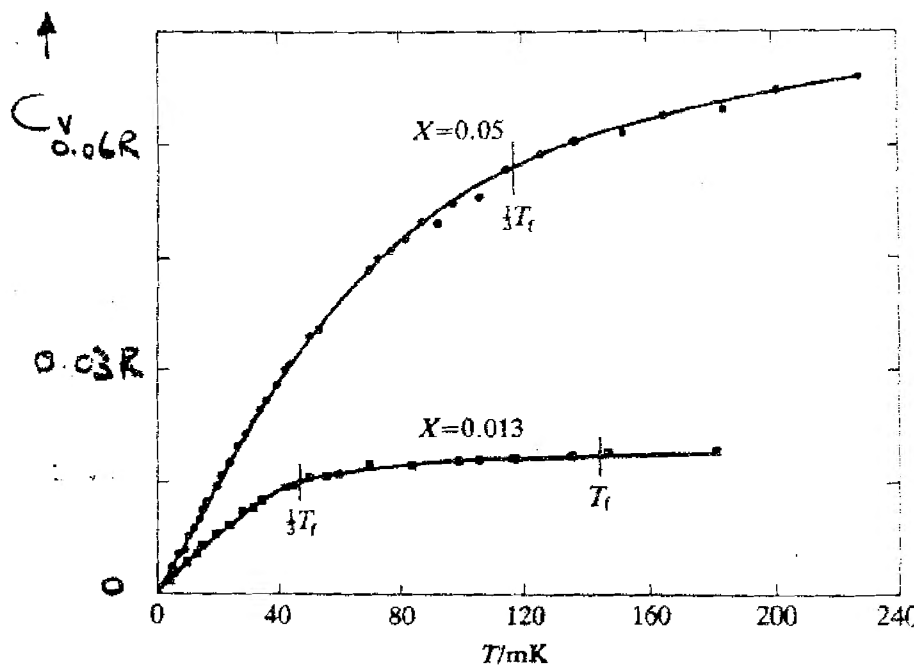
where $$X = \frac{N_3}{N_3 + N_4}$$

Liquid He^3/He^4 mixtures.



$$X = \frac{N_3}{N_3 + N_4}$$





with N_3 = number He^3 atoms in sample

N_4 = " He^4 " " "

(ii) λ line boundary between He^4 I and He^4 II – mixing in of He^3 causes decrease in temperature for line as X (proportion of He^3) increases

(iii) Above λ line mixture behaves as ordinary liquid.

(iv) In region of He^3/He^4 II mixture – for given X get usual change in He^4 II from normal $\rightleftarrows$ superfluid as temperature increases / decreases.

(v) In two phase region — no homogeneous mixture — liquid separates into He^3 rich liquid floating on denser He^4 rich liquid. Proportions of He^3 in He^3 rich and He^4 rich components vary with temperature.

Illustration on phase diagram.

For $T = 0.40$ K

He^4 rich component X has 19% He^3

He^3 rich component Y has 90% He^3

Situation similar to saturated vapour phase in liquefying gas phase diagram — see sheet.

(vi) For temperatures $T \lesssim 0.2\,K$ He^3 rich phase is $\sim 100\%$ He^3 but He^4 rich phase still contains $\sim 6\%$ He^3.

Dilute He^3 / He^4 $\underline{II}$ liquid mixtures.

In region $X < 0.15$ as T decreases He^3 atoms separated by increasing proportion of superfluid He^4 atoms

Interactions are weak v.d. Waals — effect on He^3 can be treated by effective mass adjustment

Creates system of weakly interacting fermions (He^3 atoms) in a sea of He^4 bosons

Good approximation to Fermi gas with

Fermi energy μ (or temperature T_F) given by

$$\mu = kT_f = \left(\frac{\hbar^2}{2m^*}\right)\left(3\pi^2 \frac{N_3}{V}\right)^{2/3}$$

where $\frac{N_3}{V}$ = number He^3 atoms m^{-3}

m^* = effective mass of He^3 atom

(expt^{l} value $m^* = 2.4 \times$ actual mass.)

For such mixture

T_f can be adjusted by changing He^3

concentration $\left(\frac{N_3}{V}\right)$

For $T > T_f$ consider Fermi gas in classical state

$T < T_f$ " " " " quantum state

— shown on phase diagram.

Check Fermi gas prediction by measuring C_V versus T for mixture of given proportion X over temp range that covers T_f.

Recall.

Quantum Fermi gas $\qquad C_V = \gamma T$

Classical Fermi gas $\qquad C_V = \frac{3}{2} N k$

(N = number of atoms)

Plot of experiment — see sheet.

For mixtures $X = 0.013$ and $X = 0.05$ see

(a) for $T < T_f \qquad C_V \propto T$

(b) for $T > T_f \qquad C_V$ saturates at $\frac{3}{2} R X$

(done / mol)

Excellent agreement with Fermi gas theory.

Flow characteristics.

In flow expt^s on He^3/He^4 II liquid mixtures He^3 atoms move with normal component of He^4 — do not take part in superfluid viscosity free flow.

Much more expt^s done
Reported in An Introduction to Liquid Helium
J Wilks and D S Betts.

Superconductivity.

Essentials only.

Experimental results

Deductions.

Theory

Flux quantisation

Type I / Type II superconductors

New materials.

Experimental results

1. Zero resistivity below critical temperature T_c

In 1911 found that in some metals d.c resistivity dropped suddenly to zero at critical temperature T_c

Graph ρ vs T for Hg } see
Table of values of T_c } sheet.

Points.

(i) In superconducting state $T < T_c$ – no detectable resistance

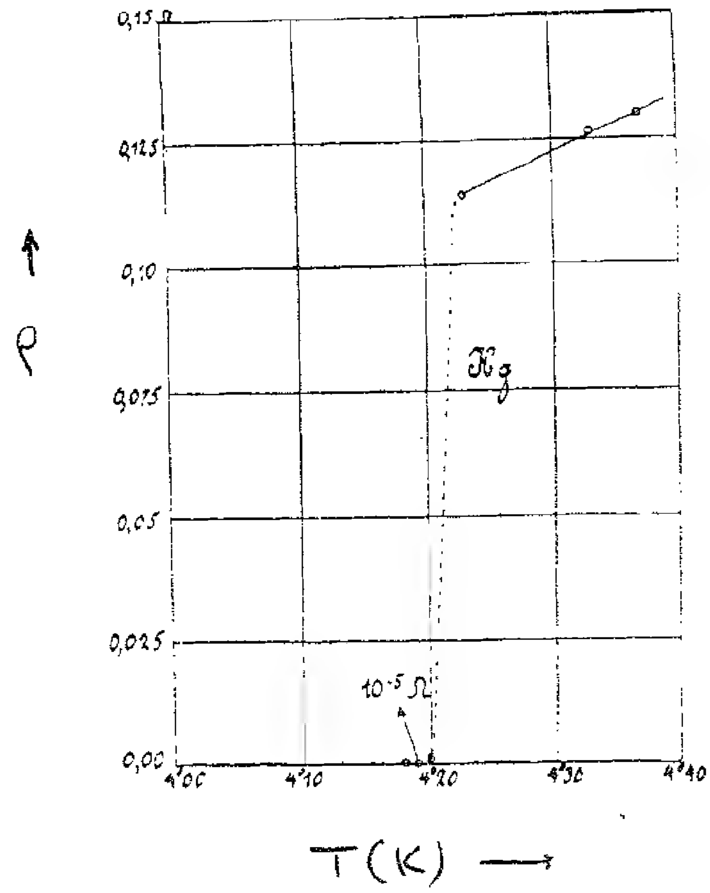
(ii) In metals and alloys – low temperature effect – see table.

(iii) Metals Cu, Ag, Au with good normal state electrical conductivity do not show superconductivity.

2. Superconductivity destroyed by strong magnetic field

Phase diagram of superconducting / normal state – see sheet.

Superconductivity



Kamerlingh Onnes discovery of superconductivity.

Superconducting transition temperature (in K) and Critical fields (at T=0) in gauss (10^{-4} T)

Transition temperature in K
Critical magnetic field at absolute zero in gauss (10^{-4} tesla)

Li	Be 0.026											B	C	N	O	F	Ne
Na	Mg											Al 1.140 105	Si*	P*	S*	Cl	Ar
K	Ca	Sc	Ti 0.39 100	V 5.38 1420	Cr*	Mn	Fe	Co	Ni	Cu	Zn 0.875 53	Ga 1.091 51	Ge*	As*	Se*	Br	Kr
Rb	Sr	Y*	Zr 0.546 47	Nb 9.50 1980	Mo 0.92 95	Tc 7.77 1410	Ru 0.51 70	Rh .0003 .049	Pd	Ag	Cd 0.56 30	In 3.4035 293	Sn (w) 3.722 309	Sb*	Te*	I	Xe
Cs*	Ba*	La fcc 6.00 1100	Hf 0.12	Ta 4.483 830	W 0.012 1.07	Re 1.4 198	Os 0.655 65	Ir 0.14 19	Pt	Au	Hg (α) 4.153 412	Tl 2.39 171	Pb 7.193 803	Bi*	Po	At	Rn
Fr	Ra	Ac															

Ce*	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu 0.1
Th 1.368 1.62	Pa 1.4	U*(α)	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

(162)

Table of values of critical field H_c - see sheet.

Experiment shows that at temperature $T < T_c$ transition between superconducting and normal states occur at field $H_c(T)$ where

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

3. Meissner - Ochsenfeld effect.

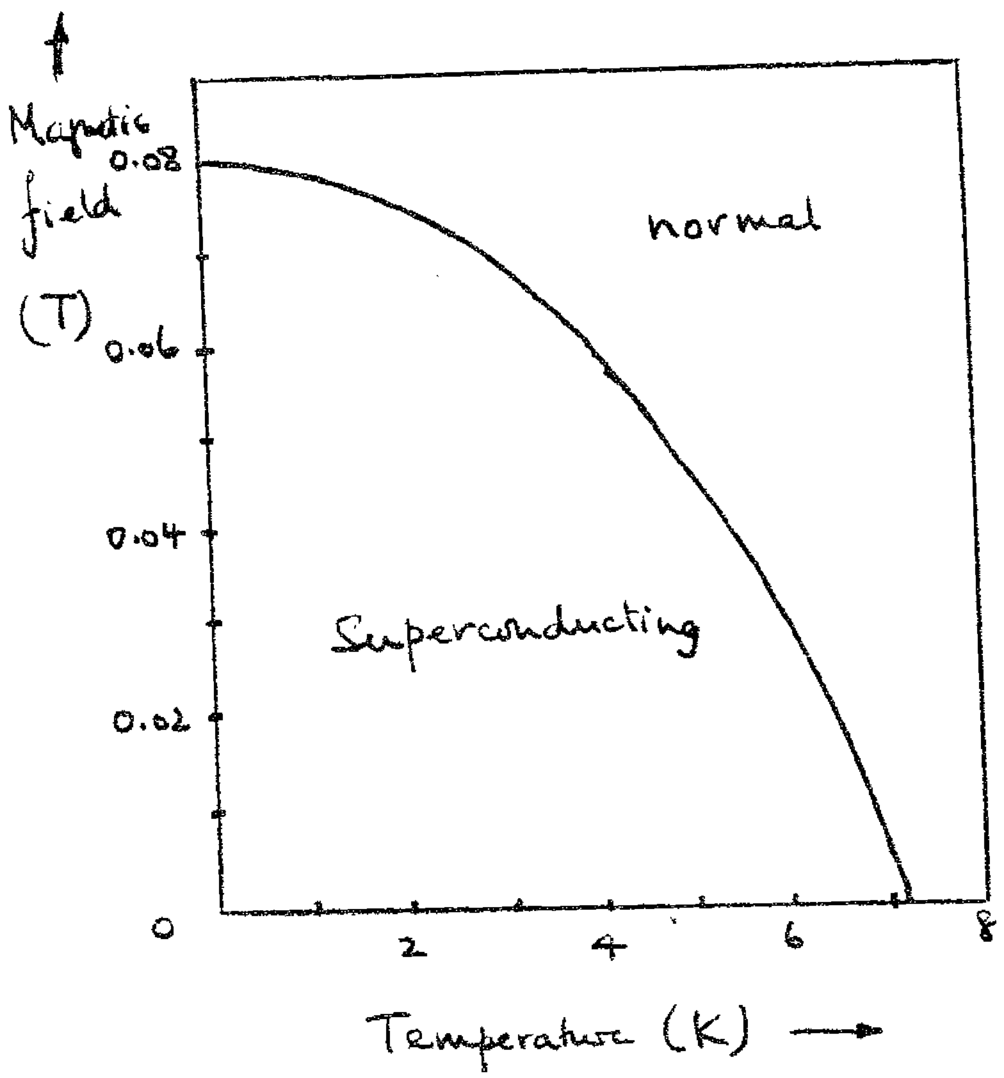
Diagram - see sheet.

Magnetic field $H < H_c$ applied to cylindrical sample.

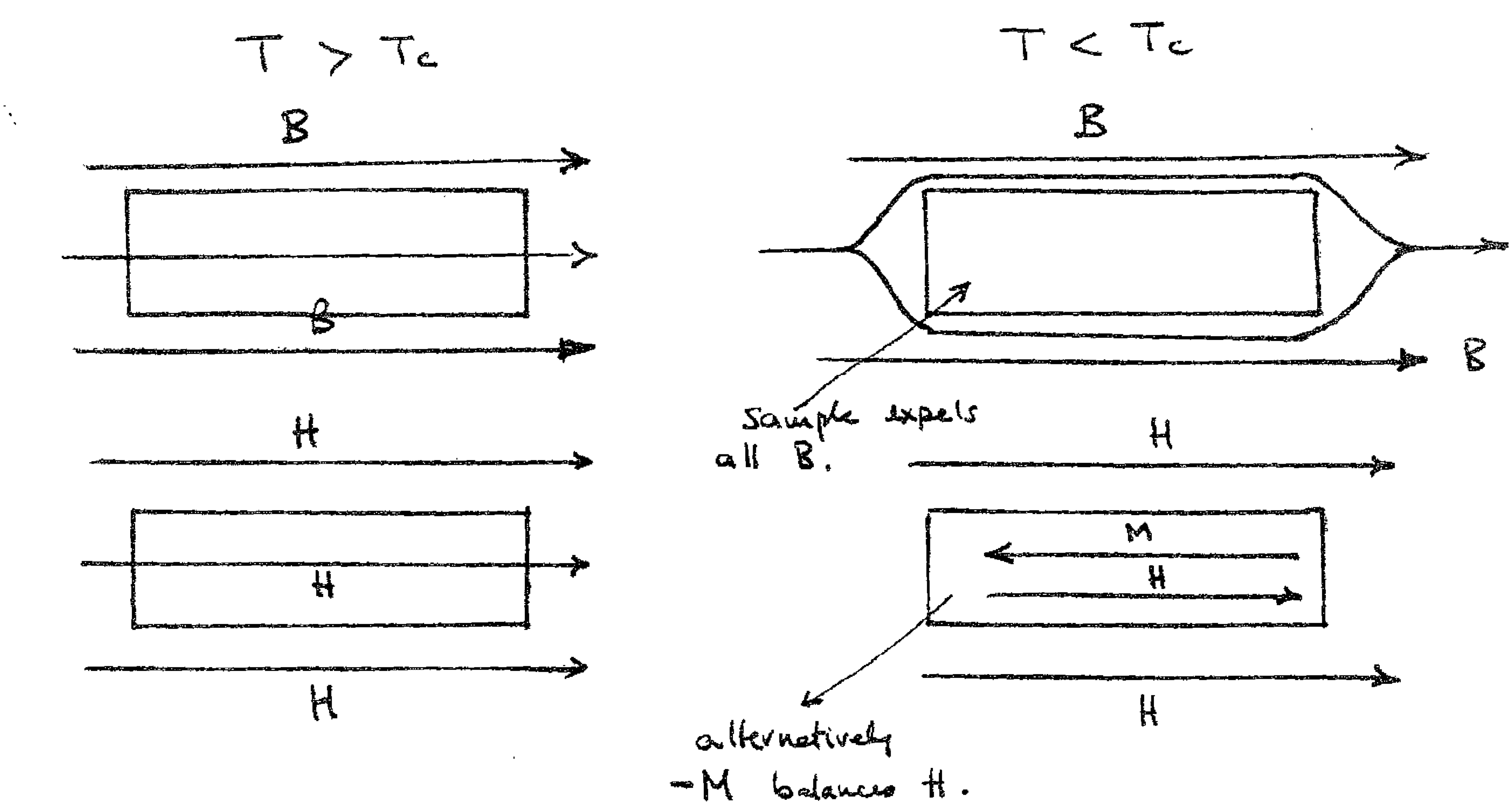
Sample cooled to below critical temperature

Superconducting state expels all magnetic flux from sample volume

Phase diagram – superconducting / normal states for Pb.



Meissner – Ochsenfeld effect.



Points.

(i) Not a simple consequence of zero electrical resistance

(ii) In superconducting volume

$$B = \underbrace{\mu_0 (H}_{\text{applied field}} + \underbrace{M)}_{\text{sample magnetisation}} = 0 \leftarrow \text{no flux in sample.}$$

Regard this as $M = -H$

Magnetic susceptibility $\chi = \dfrac{M}{H} = -1$

Said to behave as perfect diamagnet.

Graph of $-M$ vs H – see sheet.

(iii) Flux exclusion due to superconducting current flow around curved surface of cylinder – see sheet.

Current (and field) exist within penetration depth ~ 50 nm into sample.

4. Electron heat capacity in superconducting and normal state

Graphs of

Electron heat capacity C_v vs T } see sheet
Electron entropy S vs T

Points

(i) Graphs related by $C_v = T\left(\frac{dS}{dT}\right)$

(ii) Can measure values for normal state in temperature range $0 < T < T_c$ by applying magnetic field $H > H_c$

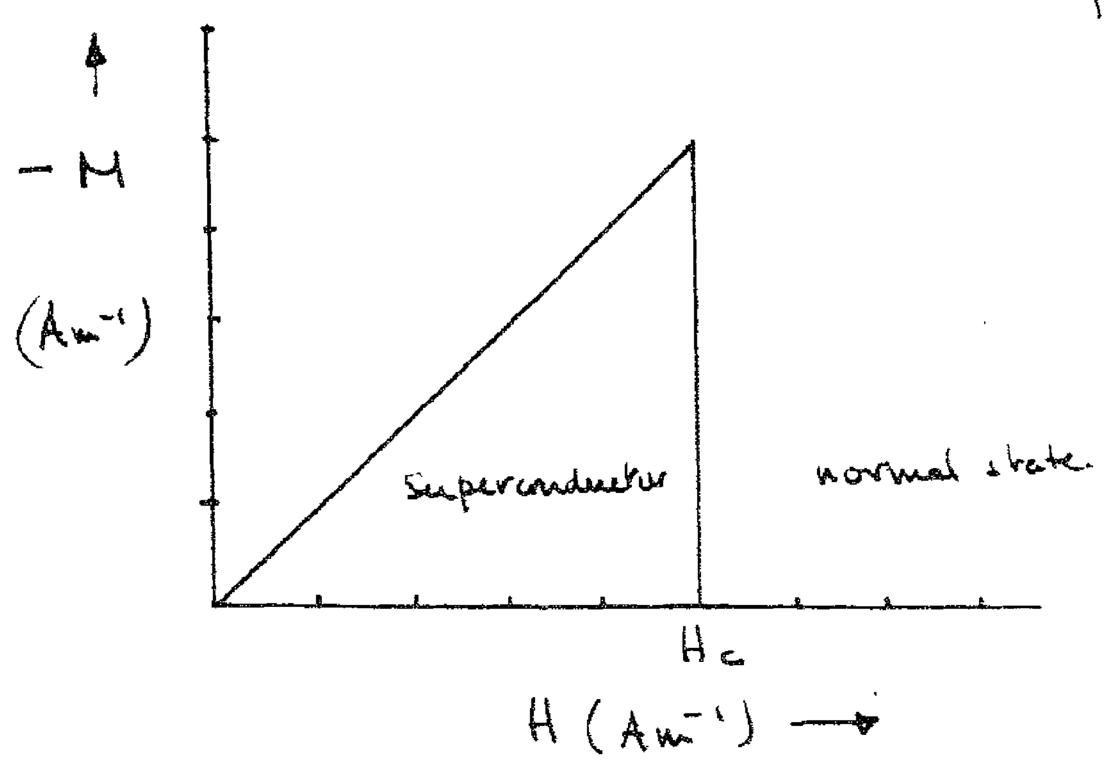
(iii) Lattice vibration contribution to C_v essentially same for normal and super states

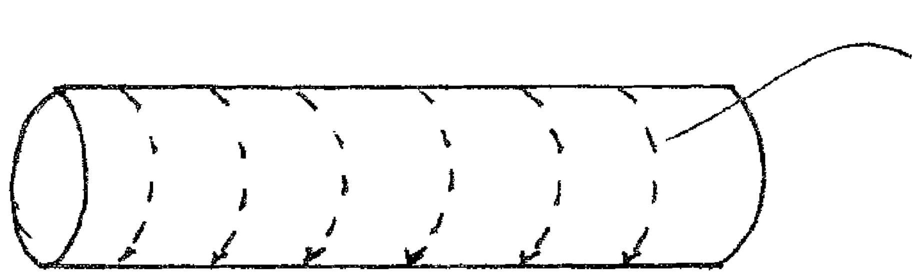
Meissner - Ochsenfeld effect.

For $H < H_c$

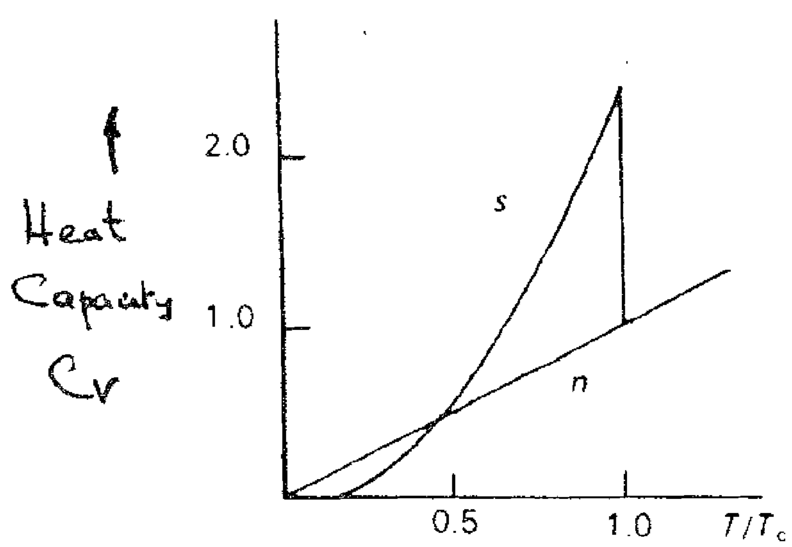
Perfect diamagnet.

$$\chi = \frac{M}{H} = -1$$

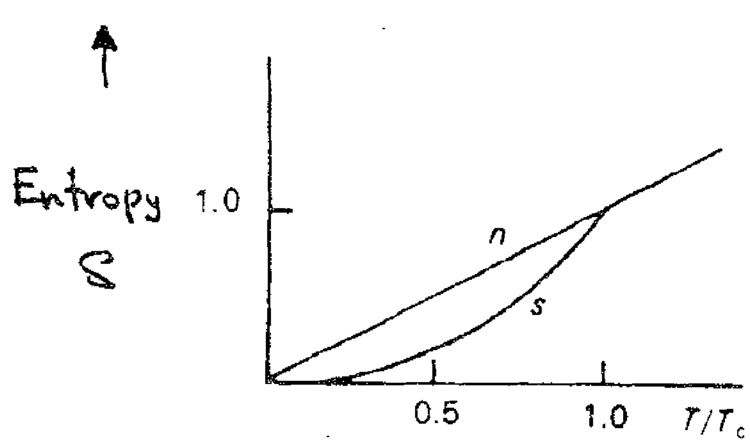




Heat capacity C_v vs T for normal and superconducting states



Entropy S vs T for normal and superconduct states.



Lattice contributions subtracted from total to give above graphs of electron C_v.

(iv) Superconducting electron heat capacity $C_v(es)$ well described by

$$C_v(es) = A \gamma T_c \exp\left(-\frac{a T_c}{T}\right)$$

where γ is constant for normal heat capacity

A and a are constants.

For Gallium $A = 7.5$ and $a = 1.4$.

Form of equation with

$$C_v \propto \exp\left(-\frac{\Delta}{2kT}\right)$$

with $\Delta = 2akT_c$

implies energy gap Δ between ground and excited states

Many other experiments.

Deductions.

1. Results suggest a two component model similar to liquid He^4 II

In superconductor

Supercurrent – flows without resistance – like superfluid He^4 II

Normal current – " with " – like normal He^4 II

2. Superconducting current carries have no entropy – make no contribution to C_v

3. Proportions of supercurrent and normal electrons governed by energy gap Δ so for $T < T_c$

Normal and Superconducting electron concentration vary as.

Normal $n \propto \exp(-\Delta/kT)$

Superconducting $s \propto [1 - \exp(-\Delta/kT)]$

C_V results indicate energy gap at $T \to 0$

$$\Delta(0) \sim 3kT_c$$

but experiment also shows that Δ depends on temperature – decreases to zero as $T \to T_c$.

Problem

In two fluid model of liquid He^4 II – Superfluid is boson condensate – in metal current is carried by electrons (fermions)

Theory.

1. Breakthrough Idea.

Pairs of electrons can couple to have zero or integer angular momentum and thus become bosons.

The pairs can condense into the ground state and carry the superconducting current.

2. Mechanism of pairing interaction (Frohlich)

Mechanism is electron - phonon - electron interaction

Description

Electron 1 distorts the lattice such that electron 2 can lower its

potential energy by following the distortion.

Electrons are coupled via the lattice distortion.

Theory developed as

Electron scatters from lattice creating a phonon. Phonon is absorbed by second electron scattering from lattice. The two electrons are coupled by creation and absorption of virtual phonons.

Full theory – BCS $\begin{cases} \text{Bardeen} \\ \text{Cooper} \\ \text{Schreiffer} \end{cases}$

not simple

describe main points without formal theory

Main points of BCS theory

1. Electron pairs (Cooper pairs) have electron wavevectors $\underline{k}$ and $-\underline{k}$ forming an $L = 0$ space w/f for pair. Spin part of w/f has $\uparrow \downarrow$ $S = 0$ character giving

$$\text{Cooper pair w/f} = \psi(1,2) = \underbrace{\psi(L=0)}_{\text{symmetric}}\,\underbrace{\chi(S=0)}_{\text{antisymm}}$$

Overall w/f is antisymmetric under particle exchange. (made of fermions)

but has total angular momentum J

$$\underline{J} = \underline{L} + \underline{S} = 0 + 0 = \underset{\text{boson.}}{0}$$

and pair acts as a boson.